

A Supersymmetric $Sp_L \times U_Y(1)$ Model

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Abstract

A supersymmetric $Sp_L(6) \times U_Y(1)$ model (SUSY $Sp(6)$) is proposed as an extension of the standard electroweak model. The model is applied in a phenomenological study of $B_d^0 \bar{B}_d^0$ mixing. It is found that the supersymmetric (SUSY) partner \tilde{z}' of the extra Z' can significantly cancel the other contributions to bring the mixing parameter x_d within the experimentally allowed range $0.57 \lesssim x_d \lesssim 0.77$ for a top mass of $158 \lesssim m_t \lesssim 194$ Gev. Other interesting and possibly novel features of flavor changing neutral currents (FCNC) in SUSY theories with horizontal gauge symmetries are pointed out.

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1 Introduction

Flavor changing neutral currents (FCNC) pose very stringent tests to extensions of the standard model (SM). They can severely limit and sometimes rule out these models (as in the case of some technicolor models).

One of the most elegant and interesting class of extensions of the standard model are SUSY models. The recent result [6] (among other things) of the unification of the couplings in a SUSY GUT but not in an ordinary GUT has renewed the confidence of theorists in SUSY models. Needless to say, SUSY theories have had no serious problems with FCNC's.

Motivated by the viability of SUSY models and the successful phenomenological studies that have been done on the $Sp_L(6) \times U_Y(1)$ model [2], [3], we propose a SUSY $Sp(6)$ model. The model is developed in analogy to the formulation of the minimal supersymmetric standard model (MSSM) from the SM [8], [9].

In section 2, we give a brief introduction of the $Sp(6)$ model. We then proceed to supersymmetrize this model by writing down the particle spectrum and a workable supersymmetric $SU_C(3) \times Sp_L(6) \times U_Y(1)$ gauge invariant lagrangian.

Section 3 examines the phenomenological consequence of SUSY $Sp(6)$ on $B_d^0 \bar{B}_d^0$ mixing. Since the gluino contributions in the MSSM still hold in SUSY $Sp(6)$, we discuss these first. We carefully discuss the assumptions made and the renormalization-group-modified rotation matrices of the squark fields. We then give the explicit expression for the mixing parameter x_d in SUSY $Sp(6)$. A plot of x_d versus the top mass m_t is made comparing the SM, MSSM, and the SUSY $Sp(6)$ results. It is found that the SM and the MSSM may be a bit too high for $0.57 \lesssim x_d \lesssim 0.77$ with a top mass $158 \lesssim m_t \lesssim 194$ Gev. SUSY $Sp(6)$ however, introduces a cancellation due to the \tilde{z}' which makes x_d fall relatively well within the experimentally allowable range for a large top mass. A discussion of further implications follows.

Section 4 gives our conclusions and outlook.

2 The Supersymmetric $SU_C(3) \times Sp_L(6) \times U_Y(1)$ Model

The $SU_C(3) \times Sp_L(6) \times U_Y(1)$ model ($Sp(6)$ model) was proposed in 1984 [1] to address the generation problem of particle physics. A common approach to introduce a “horizontal” group to tackle the generation problem is the formation of the gauge group

$$G \times SU_C(3) \times SU_L(2) \times U_Y(1)$$

where G is the horizontal group and $SU_C(3) \times SU_L(2) \times U_Y(1)$ is the familiar SM. This, however, is not very appealing since it increases the arbitrariness of the theory by adding another gauge coupling due to G . In addition, these models tend to necessitate the introduction of more fermions for anomaly cancellation which may also be questionable since experiment indicates the existence of only three light fermion families.

The $Sp(6)$ group, however, has the unique feature of unifying G and $SU_L(2)$ into a single horizontal group $Sp_L(6)$ without introducing extra fermions into the theory for anomaly cancellation since $Sp(6)$ is anomaly-free. $Sp(6)$ has a horizontal subgroup $SU_H(3)$ which mixes the different generations. In this extension, $Sp(6)$ has a six dimensional representation $\underline{6}$, which contains the six leptons and quarks in a multiplet. $Sp(6)$ decomposes into three $\underline{2}$ of $SU(2)$ which gives rise to the doublets in the three generations of leptons and quarks.

The Lie group $Sp(6)$ has 21 generators, $T^{(i)}$, $i = 1, 2, \dots, 21$ whose 6×6 representation are given by [2],

$$\frac{1}{2}\sqrt{\frac{1}{2}}(\sigma_1, \sigma_2, \sigma_3) \otimes \lambda_S^i \quad , \quad \lambda_S^i = \lambda^0, \lambda^1, \lambda^3, \lambda^4, \lambda^6, \lambda^8 \quad (1)$$

$$\frac{1}{2}\sqrt{\frac{1}{2}}\mathbf{1} \otimes \lambda_A^i \quad , \quad \lambda_A^i = \lambda^2, \lambda^5, \lambda^7 \quad (2)$$

where the σ_i , $i = 1, 2, 3$ and λ^i , $i = 0, 1, \dots, 8$ are the Pauli and Gell-Mann matrices respectively. For future reference we assign arbitrarily $T^{(i)}$ to equations 1 and 2 in equation 82 of the appendix. The normalization in equations 1 and 2 are such that

$$Tr(T^{(i)}T^{(j)}) = \frac{1}{2}\delta^{ij} \quad i, j = 1, 2, \dots, 21. \quad (3)$$

The three $SU(2)$ subgroups (which we denote by $SU_{i'}(2)$, $i' = 1, 2, 3$) to which $Sp(6)$ decomposes have the following generators

$$\vec{\Sigma}_1 \equiv \frac{1}{2}\vec{\sigma} \otimes \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix} ; \quad \vec{\Sigma}_2 \equiv \frac{1}{2}\vec{\sigma} \otimes \begin{pmatrix} 0 & & \\ & 1 & \\ & & 0 \end{pmatrix} ; \quad \vec{\Sigma}_3 \equiv \frac{1}{2}\vec{\sigma} \otimes \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}. \quad (4)$$

In the symmetry breaking scheme,

$$\begin{aligned} Sp(6) &\longrightarrow SU_1(2) \times SU_2(2) \times SU_3(2) \\ &\longrightarrow SU_{12}(2) \times SU_3(2) \\ &\longrightarrow SU_{123}(2) = SU_L(2) \end{aligned} \quad (5)$$

$SU_{12}(2)$ and $SU_{123}(2)$ are diagonal $SU(2)$ subgroups of the relevant direct product groups. As indicated in relation 5, the group $SU_{123}(2)$ is to be identified with the $SU_L(2)$ of the SM. If we denote (with space-time indices suppressed) $\vec{A} \equiv (A_1, A_2, A_3)$ to be the $SU_L(2)$ gauge bosons, and $\vec{A}^{(i')}$ with $i' = 1, 2, 3$ to be the gauge bosons associated with the three $SU_{i'}(2)$ subgroups in the symmetry breaking scheme of relation 5, we have

$$\vec{A} = \frac{1}{\sqrt{3}} (\vec{A}^{(1)} + \vec{A}^{(2)} + \vec{A}^{(3)}) \quad (6)$$

Equation 6 indicates why the $SU_L(2)$ gauge bosons couple universally to the three generations and it implies that

$$g_2 = \frac{1}{\sqrt{3}} g_{sp} \quad (7)$$

where g_2 and g_{sp} are the $SU_L(2)$ and $Sp_L(6)$ gauge coupling constants respectively. The other set of relatively light new gauge bosons are

$$(W'_1, W'_2, Z') = \frac{1}{\sqrt{6}} (\vec{A}^{(1)} + \vec{A}^{(2)} - 2\vec{A}^{(3)}) \quad (8)$$

$$(W''_1, W''_2, Z'') = \frac{1}{\sqrt{2}} (\vec{A}^{(1)} - \vec{A}^{(2)}) \quad (9)$$

From equations 8 and 9, it is evident that these gauge bosons do not couple universally to the three generations. The lightest of these extra gauge bosons which can possibly be detected in the near future is the neutral gauge boson Z' .

To get the coupling of the Z' with the fermions, we first write the term in the $Sp(6)$ model lagrangian describing the kinetic energy of the matter (fermion) fields and gauge-matter interactions (see the appendix for notation, conventions and some relevant formulas).

$$\begin{aligned} \mathcal{L}_{kin} = & i\bar{\Psi}'_{(i_6)L}\gamma_\mu \mathcal{D}^{(2)\mu}\Psi'_{(i_6)L} + i\bar{\Psi}_{rt}^I\gamma_\mu \mathcal{D}^{(1)\mu}\Psi_{rt}^I + \\ & i(\bar{\Psi}'_Q)_{(i_6)\alpha L}\gamma_\mu \nabla^\mu(\Psi'_Q)_{(i_6)\alpha L} + i(\bar{\Psi}_u)^I_{\alpha rt}\gamma_\mu \mathcal{D}^{(2)\mu}(\Psi_u)^I_{\alpha rt} + \\ & i(\bar{\Psi}_d)^I_{\alpha rt}\gamma_\mu \mathcal{D}^{(2)\mu}(\Psi_d)^I_{\alpha rt} \end{aligned} \quad (10)$$

In this paper, primed fermion (and later sfermion) fields are the initial fields (as opposed to the physical, mass eigenstate fields).

As an example of extracting the Z' -quark interaction terms, let us rewrite the third term in equation 10. Using equation 100,

$$\begin{aligned} i(\Psi'_Q)_{(i_6)\alpha L}\gamma^\mu & \left[\partial^\mu(\Psi'_Q)_{(i_6)\alpha L} + ig_3 G_a^\mu Y_{\alpha\beta}^a(\Psi'_Q)_{(i_6)\beta L} + ig_{sp} A_j^\mu T_{i_6 j_6}^{(j)}(\Psi'_Q)_{(j_6)\alpha L} \right. \\ & \left. + i\frac{g_1}{2} B^\mu y(\Psi'_Q)_{(i_6)\alpha L} \right] \end{aligned} \quad (11)$$

Looking at the term

$$i(\bar{\Psi}'_Q)_{(i_6)\alpha L}\gamma_\mu \left[ig_{sp} A_j^\mu T_{i_6 j_6}^{(j)}(\Psi'_Q)_{(j_6)\alpha L} \right] \quad (12)$$

we have to identify which A_j^μ will correspond to Z' (or possibly a linear combination of A_j^μ). From equation 8,

$$Z' = \frac{1}{\sqrt{6}} (\vec{A}_3^{(1)} + \vec{A}_3^{(2)} - 2\vec{A}_3^{(3)}) \quad (13)$$

We then have to get the relation of $(\vec{A}^{(1)})^\mu, (\vec{A}^{(2)})^\mu, (\vec{A}^{(3)})^\mu$ with A_j^μ of $Sp(6)$. The key point to realize here is that these three sets of gauge bosons associated with the three $SU_{(i')}(2)$ groups would then correspond to the three sets of generators in equation 4 in the following manner,

$$(\vec{A}^{(i')})^\mu \longleftrightarrow \vec{\Sigma}_{i'}, \quad i' = 1, 2, 3 \quad (14)$$

and also

$$A_j^\mu \longleftrightarrow T^{(j)}. \quad (15)$$

Hence from equations 14 and 15, if we can write $T^{(j)}$ as linear combinations of $\vec{\Sigma}_{i'}$, then we could write A_j^μ as linear combinations of $(\vec{A}^{(i')})^\mu$ and vice versa. Knowing the expressions of $(\vec{A}^{(i')})^\mu$ in terms of the A_j^μ of $Sp(6)$, we can then put these expressions into 13. Going through these steps, we find

$$Z'_\nu = A_{\nu(18)} \quad (16)$$

where (see also the appendix equation 82)

$$A_{\nu(18)} \longleftrightarrow T^{(18)} = \frac{1}{2\sqrt{2}}\sigma_3 \otimes \lambda^8 \quad (17)$$

Hence, to get the term in equation 12 describing the interaction of the Z' with the left handed quarks we look at the term

$$i(\bar{\Psi}'_Q)_{(i_6)\alpha L}\gamma_\mu \left[ig_{sp} A_{(18)}^\mu T_{i_6 j_6}^{(18)} (\Psi'_Q)_{(j_6)\alpha L} \right] = i(\bar{\Psi}'_Q)_{(i_6)\alpha L}\gamma_\mu \left[ig_{sp} Z'^\mu T_{i_6 j_6}^{(18)} (\Psi'_Q)_{(j_6)\alpha L} \right]. \quad (18)$$

Of course, one then has to express the initial (primed) fields in terms of the physical fields by rotating the initial fields using the appropriate unitary matrices.

Let us now describe the (minimal) SUSY $Sp(6)$ model. To establish notation, let us list down the particle spectrum of SUSY $Sp(6)$ in tables 1, 2, 3, 4 and 5. We also list down their quantum numbers in table 6.

Table 1: Gauge Bosons and Gauginos in SUSY $Sp(6)$

| vector superfields | bosonic components | fermionic components | auxiliary fields |
|-------------------------------------|-----------------------|-------------------------|---------------------|
| \mathcal{G}^a (for $SU_C(3)$) | G_μ^a | λ_G^a | D_G^a |
| \mathcal{A}^i (for $Sp_L(6)$) | A_μ^i | λ_A^i | D_A^i |
| $\hat{\mathcal{B}}$ (for $U_Y(1)$) | B_μ | λ_B | D_B |

Note that in the tables, the fermionic components are two-component spinors. For the chiral superfields, we denote these two-component spinors by the lower case greek letter ψ (as opposed to the usual four-component Dirac spinor as in equation 10 which we denote by the upper case greek letter Ψ). The complete set of formulas for converting two-component to four-component spinors are given in equations (A19) to (A23) of reference [7]. There, Ψ_i is defined as

$$\Psi_i = \begin{pmatrix} \xi_i \\ \bar{\eta}_i \end{pmatrix} \quad (19)$$

Table 2: Leptons and Sleptons in SUSY $Sp(6)$

| chiral superfields | bosonic components | fermionic components |
|--|--|--|
| $\mathcal{L}_{(i_6)} = \begin{bmatrix} \mathcal{L}_1 \\ \mathcal{L}_2 \\ \mathcal{L}_3 \\ \mathcal{L}_4 \\ \mathcal{L}_5 \\ \mathcal{L}_6 \end{bmatrix}$ | $\begin{bmatrix} L'_1 \\ L'_2 \\ L'_3 \\ L'_4 \\ L'_5 \\ L'_6 \end{bmatrix} = \begin{bmatrix} \tilde{\nu}'_{eL} \\ \tilde{\nu}'_{\mu L} \\ \tilde{\nu}'_{\tau L} \\ \tilde{e}'_L \\ \tilde{\mu}'_L \\ \tilde{\tau}'_L \end{bmatrix}$ | $\begin{bmatrix} \psi'_{l1} \\ \psi'_{l2} \\ \psi'_{l3} \\ \psi'_{l4} \\ \psi'_{l5} \\ \psi'_{l6} \end{bmatrix} = \begin{bmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \\ e'_L \\ \mu'_L \\ \tau'_L \end{bmatrix}$ |
| \mathcal{R}^1 | $R'^1 = \tilde{e}_R'^{+1} = \tilde{e}_R'^+$ | $\psi_R'^1 = (e_L'^1)^c = e_L'^c$ |
| \mathcal{R}^2 | $R'^2 = \tilde{e}_R'^{+2} = \tilde{\mu}_R'^+$ | $\psi_R'^2 = (e_L'^2)^c = \mu_L'^c$ |
| \mathcal{R}^3 | $R'^3 = \tilde{e}_R'^{+3} = \tilde{\tau}_R'^+$ | $\psi_R'^3 = (e_L'^3)^c = \tau_L'^c$ |

 Table 3: Quarks and Squarks in SUSY $Sp(6)$

| chiral superfields | bosonic components | fermionic components |
|--|---|---|
| $\mathcal{Q}_{(i_2)\alpha} = \begin{bmatrix} \mathcal{Q}_{1\alpha} \\ \mathcal{Q}_{2\alpha} \\ \mathcal{Q}_{3\alpha} \\ \mathcal{Q}_{4\alpha} \\ \mathcal{Q}_{5\alpha} \\ \mathcal{Q}_{6\alpha} \end{bmatrix}$ | $\begin{bmatrix} Q'_{1\alpha} \\ Q'_{2\alpha} \\ Q'_{3\alpha} \\ Q'_{4\alpha} \\ Q'_{5\alpha} \\ Q'_{6\alpha} \end{bmatrix} = \begin{bmatrix} \tilde{u}'_{L\alpha} \\ \tilde{c}'_{L\alpha} \\ \tilde{t}'_{L\alpha} \\ \tilde{d}'_{L\alpha} \\ \tilde{s}'_{L\alpha} \\ \tilde{b}'_{L\alpha} \end{bmatrix}$ | $\begin{bmatrix} \psi'_{q1\alpha} \\ \psi'_{q2\alpha} \\ \psi'_{q3\alpha} \\ \psi'_{q4\alpha} \\ \psi'_{q5\alpha} \\ \psi'_{q6\alpha} \end{bmatrix} = \begin{bmatrix} u'_{L\alpha} \\ c'_{L\alpha} \\ t'_{L\alpha} \\ d'_{L\alpha} \\ s'_{L\alpha} \\ b'_{L\alpha} \end{bmatrix}$ |
| $\mathcal{D}_\alpha^1 = \mathcal{D}_\alpha$ | $D_\alpha'^1 = \tilde{d}_{R\alpha}'^{*1} = \tilde{d}_{R\alpha}'^*$ | $\psi_{D\alpha}'^1 = (d_{L\alpha}'^1)^c = d_{L\alpha}'^c$ |
| $\mathcal{D}_\alpha^2 = \mathcal{S}_\alpha$ | $D_\alpha'^2 = \tilde{d}_{R\alpha}'^{*2} = \tilde{s}_{R\alpha}'^*$ | $\psi_{D\alpha}'^2 = (d_{L\alpha}'^2)^c = s_{L\alpha}'^c$ |
| $\mathcal{D}_\alpha^3 = \mathcal{B}_\alpha$ | $D_\alpha'^3 = \tilde{d}_{R\alpha}'^{*3} = \tilde{b}_{R\alpha}'^*$ | $\psi_{D\alpha}'^3 = (d_{L\alpha}'^3)^c = b_{L\alpha}'^c$ |
| $\mathcal{U}_\alpha^1 = \mathcal{U}_\alpha$ | $U_\alpha'^1 = \tilde{u}_{R\alpha}'^{*1} = \tilde{u}_{R\alpha}'^*$ | $\psi_{U\alpha}'^1 = (u_{L\alpha}'^1)^c = u_{L\alpha}'^c$ |
| $\mathcal{U}_\alpha^2 = \mathcal{C}_\alpha$ | $U_\alpha'^2 = \tilde{u}_{R\alpha}'^{*2} = \tilde{c}_{R\alpha}'^*$ | $\psi_{U\alpha}'^2 = (u_{L\alpha}'^2)^c = c_{L\alpha}'^c$ |
| $\mathcal{U}_\alpha^3 = \mathcal{T}_\alpha$ | $U_\alpha'^3 = \tilde{u}_{R\alpha}'^{*3} = \tilde{t}_{R\alpha}'^*$ | $\psi_{U\alpha}'^3 = (u_{L\alpha}'^3)^c = t_{L\alpha}'^c$ |

Table 4: Higgs and Higgsinos in SUSY $Sp(6)$

| chiral superfields | bosonic components | fermionic components |
|--|---|---|
| $\hat{\Phi}_{(i_6)} = \begin{bmatrix} \hat{\Phi}_1 \\ \hat{\Phi}_2 \\ \hat{\Phi}_3 \\ \hat{\Phi}_4 \\ \hat{\Phi}_5 \\ \hat{\Phi}_6 \end{bmatrix}$ | $\phi_{(i_6)} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \end{bmatrix}$ | $\psi_{\phi(i_6)} = \begin{bmatrix} \psi_{\phi 1} \\ \psi_{\phi 2} \\ \psi_{\phi 3} \\ \psi_{\phi 4} \\ \psi_{\phi 5} \\ \psi_{\phi 6} \end{bmatrix}$ |
| $\mathcal{H}_{(i_6)} = \begin{bmatrix} \mathcal{H}_1 \\ \mathcal{H}_2 \\ \mathcal{H}_3 \\ \mathcal{H}_4 \\ \mathcal{H}_5 \\ \mathcal{H}_6 \end{bmatrix}$ | $H_{(i_6)} = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \\ H_6 \end{bmatrix}$ | $\psi_{H(i_6)} = \begin{bmatrix} \psi_{H1} \\ \psi_{H2} \\ \psi_{H3} \\ \psi_{H4} \\ \psi_{H5} \\ \psi_{H6} \end{bmatrix}$ |

Table 5: Auxiliary Fields of the Matter Multiplets in SUSY $Sp(6)$

| chiral superfields | auxiliary fields |
|-----------------------------|---------------------|
| $\mathcal{L}_{(i_6)}$ | $F_{l(i_6)}$ |
| \mathcal{R}^I | F_R^I |
| $\mathcal{Q}_{(i_6)\alpha}$ | $F_{q(i_6)\alpha}$ |
| \mathcal{D}_α^I | $F_{D\alpha}^I$ |
| \mathcal{U}_α^I | $F_{U\alpha}^I$ |
| $\hat{\Phi}_{(i_6)}$ | $F_{\phi(i_6)}$ |
| $\mathcal{H}_{(i_6)}$ | $F_{H(i_6)}$ |

Table 6: Quantum Numbers of Particles in SUSY $Sp(6)$

| superfields | $SU_C(3)$ transformation | $Sp_L(6)$ transformation | y (hypercharge) |
|-----------------------------|-----------------------------|-----------------------------|--------------------|
| \mathcal{G}^a | $\underline{8}$ | $\underline{1}$ | 0 |
| \mathcal{A}^i | $\underline{1}$ | $\underline{21}$ | 0 |
| $\hat{\mathcal{B}}$ | $\underline{1}$ | $\underline{1}$ | 0 |
| $\mathcal{L}_{(i_6)}$ | $\underline{1}$ | $\underline{6}$ | -1 |
| \mathcal{R}^I | $\underline{1}$ | $\underline{1}$ | 2 |
| $\mathcal{Q}_{(i_6)\alpha}$ | $\underline{3}$ | $\underline{6}$ | $\frac{1}{3}$ |
| \mathcal{D}_α^I | $\underline{3}$ | $\underline{1}$ | $\frac{2}{3}$ |
| \mathcal{U}_α^I | $\underline{3}$ | $\underline{1}$ | $-\frac{4}{3}$ |
| $\hat{\Phi}_{(i_6)}$ | $\underline{1}$ | $\underline{6}$ | 1 |
| $\mathcal{H}_{(i_6)}$ | $\underline{1}$ | $\underline{6}$ | -1 |

where ξ_i and η_i are two-component spinors. These conversions are indispensable in deriving Feynman rules.

The bosonic superpartners of the ordinary fermions, namely leptons and quarks are indicated by the same letter but with a tilde on top (example: if $e \longrightarrow$ electron then $\tilde{e} \longrightarrow$ selectron). As usual we refer to superpartners of fermions as sfermions, while for gauge bosons, we refer to their superpartners as gauginos.

\mathcal{G}^a , \mathcal{A}^i and $\hat{\mathcal{B}}$ are the vector superfield multiplets of the gluons, $Sp_L(6)$, and $U_Y(1)$ gauge bosons respectively and their superpartners. \mathcal{L} and \mathcal{Q} denote the superfield multiplet of left-handed leptons and quarks respectively and their superpartners while the \mathcal{R} , \mathcal{D} and \mathcal{U} on the other hand denote the superfield multiplet of right-handed electron-type leptons, down-type quarks and up-type quarks respectively and their superpartners. The two types of higgs chiral superfield multiplets are denoted by $\hat{\Phi}_{(i_6)}$ and $\mathcal{H}_{(i_6)}$. As in the MSSM, we introduce the two types of higgs to cancel the anomaly due to the superpartner of the original higgs.

Given the above particle spectrum, we can now write a (minimal) supersymmetric $SU_C(3) \times Sp_L(6) \times U_Y(1)$ gauge-invariant lagrangian given by

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_{kin} + \mathcal{L}_{superpotential} + \mathcal{L}_{soft-breaking} \quad (20)$$

where

$$\begin{aligned} \mathcal{L}_{YM} = & \frac{1}{4k_3(2g_3)^2} \text{Tr} \left[W_G^\eta W_{G\eta} \Big|_{\theta\theta} + \bar{W}_{G\dot{\eta}} \bar{W}_G^{\dot{\eta}} \Big|_{\bar{\theta}\bar{\theta}} \right] + \\ & \frac{1}{4k(2g_{sp})^2} \text{Tr} \left[W_A^\eta W_{A\eta} \Big|_{\theta\theta} + \bar{W}_{A\dot{\eta}} \bar{W}_A^{\dot{\eta}} \Big|_{\bar{\theta}\bar{\theta}} \right] + \\ & \frac{1}{4} \left[W_B^\eta W_{B\eta} \Big|_{\theta\theta} + \bar{W}_{B\dot{\eta}} \bar{W}_B^{\dot{\eta}} \Big|_{\bar{\theta}\bar{\theta}} \right] \end{aligned} \quad (21)$$

$$\begin{aligned}
\mathcal{L}_{kin} = & \mathcal{L}_{(i_6)}^\dagger e^{2\left[g_{sp} T_{i_6 j_6}^{(i)} \mathcal{A}_i + g_1 \left(\frac{1}{2}\right) (-1) \delta_{i_6 j_6} \hat{\mathcal{B}}\right]} \mathcal{L}_{(j_6)} \Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \\
& \mathcal{R}^{I\dagger} e^{2\left[g_1 \left(\frac{1}{2}\right) (2) \hat{\mathcal{B}}\right]} \mathcal{R}^I \Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \\
& \mathcal{Q}_{(i_6)\alpha}^\dagger e^{2\left[g_3 Y_{\alpha\beta}^a \mathcal{G}_a \delta_{i_6 j_6} + g_{sp} \delta_{\alpha\beta} T_{i_6 j_6}^{(i)} \mathcal{A}_i + g_1 \left(\frac{1}{2}\right) \left(\frac{1}{3}\right) \delta_{\alpha\beta} \delta_{i_6 j_6} \hat{\mathcal{B}}\right]} \mathcal{Q}_{(j_6)\beta} \Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \\
& \mathcal{D}_\alpha^{I\dagger} e^{2\left[g_3 \bar{Y}_{\alpha\beta}^a \mathcal{G}_a + g_1 \left(\frac{1}{2}\right) \left(\frac{2}{3}\right) \delta_{\alpha\beta} \hat{\mathcal{B}}\right]} \mathcal{D}_\beta^I \Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \\
& \mathcal{U}_\alpha^{I\dagger} e^{2\left[g_3 \bar{Y}_{\alpha\beta}^a \mathcal{G}_a + g_1 \left(\frac{1}{2}\right) \left(-\frac{4}{3}\right) \delta_{\alpha\beta} \hat{\mathcal{B}}\right]} \mathcal{U}_\beta^I \Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \\
& \mathcal{H}_{(i_6)}^\dagger e^{2\left[g_{sp} T_{i_6 j_6}^{(i)} \mathcal{A}_i + g_1 \left(\frac{1}{2}\right) (-1) \delta_{i_6 j_6} \hat{\mathcal{B}}\right]} \mathcal{H}_{(j_6)} \Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \\
& \hat{\Phi}_{(i_6)}^\dagger e^{2\left[g_{sp} T_{i_6 j_6}^{(i)} \mathcal{A}_i + g_1 \left(\frac{1}{2}\right) (1) \delta_{i_6 j_6} \hat{\mathcal{B}}\right]} \hat{\Phi}_{(j_6)} \Big|_{\theta\theta\bar{\theta}\bar{\theta}}
\end{aligned} \tag{22}$$

$$\begin{aligned}
\mathcal{L}_{superpotential} = & \mu \eta_{i_6 j_6} \mathcal{H}_{(i_6)} \hat{\Phi}_{(j_6)} \Big|_{\theta\theta} + g_{eI} \eta_{i_6 j_6} \mathcal{H}_{(i_6)} \mathcal{L}_{(j_6)} \mathcal{R}^I \Big|_{\theta\theta} + \\
& g_{dI} \eta_{i_6 j_6} \mathcal{H}_{(i_6)} \mathcal{Q}_{(j_6)\alpha} \mathcal{D}_\alpha^I \Big|_{\theta\theta} + g_{uI} \eta_{i_6 j_6} \hat{\Phi}_{(j_6)} \mathcal{Q}_{(i_6)\alpha} \mathcal{U}_\alpha^I \Big|_{\theta\theta} + \text{h.c.}
\end{aligned} \tag{23}$$

$$\begin{aligned}
\mathcal{L}_{soft-breaking} = & -m_H^2 H_{(i_6)}^* H_{(i_6)} - m_\phi^2 \phi_{(i_6)}^* \phi_{(i_6)} \\
& - (m_L^2) L_{(i_6)}'^* L_{(i_6)}' - (m_R^2)^{IJ} R'^{I*} R'^J \\
& - m_Q^2 Q_{(i_6)\alpha}^* Q'_{(i_6)\alpha} - (m_D^2)^{IJ} D_\alpha^{I*} D_\alpha^J - (m_U^2)^{IJ} U_\alpha^{I*} U_\alpha^J \\
& + m_1 [\lambda_G^a \lambda_G^a + \text{h.c.}] + m_2 [\lambda_A^i \lambda_A^i + \text{h.c.}] + m_3 [\lambda_B \lambda_B + \text{h.c.}] \\
& + [h \eta_{i_6 j_6} H_{(i_6)} \phi_{(j_6)} + h_{eI} \eta_{i_6 j_6} H_{(i_6)} L'_{(j_6)} R'^I \\
& + h_{dI} \eta_{i_6 j_6} H_{(i_6)} Q_{(j_6)\alpha}^I D_\alpha^I + h_{uI} \eta_{i_6 j_6} \phi_{(i_6)} Q'_{(i_6)\alpha} U_\alpha^I + \text{h.c.}]
\end{aligned} \tag{24}$$

The derivation of the preceding lagrangian was done in analogy to that of the minimal supersymmetric standard model (MSSM) [8], [9]. Some relevant formulas are found in the appendix . In equation 21,

$$k_3 \ni \text{Tr} (Y^a Y^b) = k_3 \delta^{ab}, \quad k_3 > 0 \tag{25}$$

and

$$k \ni \text{Tr} (T^{(i)} T^{(j)}) = k \delta^{ij}, \quad k > 0. \tag{26}$$

To be able to do calculations from the preceding lagrangian, we have to deal with the component fields of the superfields. Instead of writing down the expansions for all the superfield expressions in equations 21 to 23, we will just show the expansion for “prototype” structures and then other terms which have similar structures are calculated by substituting analogous quantities. These prototype structures are derived using equations 104 to 106 of the appendix , the following prototype structures (V , Φ_i are vector and chiral superfields respectively)

$$V(x) = -\theta \sigma^\mu \bar{\theta} v_\mu + i \theta \theta \bar{\theta} \bar{\lambda} - i \bar{\theta} \bar{\theta} \theta \lambda + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} D \tag{27}$$

$$\begin{aligned}
\Phi(x) &= A(y) + \sqrt{2} \theta \psi(y) + \theta \theta F(y) \\
&= A(x) + i \theta \sigma^\mu \bar{\theta} \partial_\mu A(x) + \frac{1}{4} \theta \theta \bar{\theta} \bar{\theta} \square A(x) + \\
&\quad \sqrt{2} \theta \psi(x) - \frac{i}{\sqrt{2}} \theta \theta \partial_\mu \psi(x) \sigma^\mu \bar{\theta} + \theta \theta F(x)
\end{aligned} \tag{28}$$

and the definitions,

$$D_\eta \equiv \frac{\partial}{\partial \theta^\eta} + i\sigma_{\eta\dot{\eta}}^\mu \bar{\theta}^{\dot{\eta}} \partial_\mu \quad (29)$$

$$\bar{D}_{\dot{\eta}} \equiv -\frac{\partial}{\partial \bar{\theta}^{\dot{\eta}}} - i\theta^\eta \sigma_{\eta\dot{\eta}}^\mu \partial_\mu. \quad (30)$$

One has to also take note of the fermionic, bosonic and auxiliary field components of the superfields as given in tables 1, 2, 3, 4 and 5. The auxiliary fields D_G^a, \dots , etc. and $F_{l(i_6)}, \dots$, etc. (see tables 1 and 5) are eliminated from the lagrangian through the equations of motion,

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} \right) = 0 \quad (31)$$

for a field ϕ . Note that the fundamental equations given above and their manipulation were taken from reference [10]. Since their metric tensor is different from what we use here, we have to adjust the signs of the terms involving the component fields.

In the following formulas, one will notice terms like $\left[\frac{1}{2} \cdot \frac{1}{3}\right]$ or $(\frac{1}{2})(\frac{1}{3})$. We purposely did not simplify this to emphasize the fact that we multiply $\frac{1}{2}$ by the hypercharge of the multiplet. For example, in equations 35, 36 and 37 below, we have $(\frac{1}{2})(\frac{1}{3})$ and $\left[\frac{1}{2} \cdot \frac{1}{3}\right]$ because the $\mathcal{Q}_{(i_6)}$ multiplet has a hypercharge $\frac{1}{3}$ as in table 6.

For the \mathcal{L}_{YM} , we use the prototype structure

$$\frac{1}{4k_3(2g_3)^2} \text{Tr} \left[W_G^\eta W_{G\eta} |_{\theta\theta} + \bar{W}_{G\dot{\eta}} \bar{W}_G^{\dot{\eta}} |_{\bar{\theta}\bar{\theta}} \right] = -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu a} + i\bar{\lambda}_{Ga} \bar{\sigma}^\mu [\mathcal{D}_\mu \lambda_G]_a + \frac{1}{2} D_{Ga} D_{Ga} \quad (32)$$

where

$$G_a^{\mu\nu} \equiv \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_3 f_{abc} G_b^\mu G_c^\nu \quad (33)$$

$$[\mathcal{D}_\mu \lambda_G]_a = \partial_\mu \lambda_{Ga} - g_3 f_{abc} G_{\mu b} \lambda_{Gc}. \quad (34)$$

For the \mathcal{L}_{kin} , we use the prototype structure

$$\begin{aligned} & \mathcal{Q}_{(i_6)\alpha}^\dagger e^2 \left[g_3 Y_{\alpha\beta}^a \mathcal{G}_a \delta_{i_6 j_6} + g_{sp} \delta_{\alpha\beta} T_{i_6 j_6}^{(i)} \mathcal{A}_i + g_1 \left(\frac{1}{2}\right) \left(\frac{1}{3}\right) \delta_{\alpha\beta} \delta_{i_6 j_6} \hat{\mathcal{B}} \right] \mathcal{Q}_{(j_6)\beta} \Big|_{\theta\theta\bar{\theta}\bar{\theta}} = \\ & \nabla_\mu Q_{(i_6)\gamma}^* \nabla^\mu Q'_{(i_6)\gamma} + i\bar{\psi}'_{q(i_6)\alpha} \bar{\sigma}^\mu \nabla_\mu \psi'_{q(i_6)\alpha} + F_{q(i_6)\alpha}^* F_{q(i_6)\alpha} \\ & + i\sqrt{2} g_3 Y_{\alpha\beta}^a \left[Q_{(i_6)\alpha}^* \psi'_{q(i_6)\beta} \lambda_{Ga} - \bar{\lambda}_{Ga} \bar{\psi}'_{q(i_6)\alpha} Q'_{(i_6)\beta} \right] \\ & + i\sqrt{2} g_{sp} T_{i_6 j_6}^{(i)} \left[Q_{(i_6)\alpha}^* \psi'_{q(j_6)\alpha} \lambda_{Ai} - \bar{\lambda}_{Ai} Q'_{(j_6)\alpha} \bar{\psi}'_{q(i_6)\alpha} \right] \\ & + i\sqrt{2} g_1 \left[\frac{1}{2} \cdot \frac{1}{3}\right] \left[Q_{(i_6)\alpha}^* \lambda_B \psi'_{q(i_6)\alpha} - Q'_{(i_6)\alpha} \bar{\lambda}_B \bar{\psi}'_{q(i_6)\alpha} \right] \\ & + g_3 D_{Ga} Q_{(i_6)\alpha}^* Y_{\alpha\beta}^a Q'_{(i_6)\beta} + g_{sp} D_{Ai} Q_{(i_6)\alpha}^* T_{i_6 j_6}^{(i)} Q'_{(j_6)\alpha} \\ & + g_1 D_B Q_{(i_6)\alpha}^* \left[\frac{1}{2} \cdot \frac{1}{3}\right] Q'_{(i_6)\alpha} \end{aligned} \quad (35)$$

where

$$\begin{aligned} \nabla^\mu Q'_{(i_6)\gamma} & \equiv \partial^\mu Q'_{(i_6)\gamma} + i g_3 G_b^\mu Y_{\gamma\beta}^b Q'_{(i_6)\beta} + i g_{sp} A_j^\mu T_{i_6 j_6}^{(j)} Q'_{(j_6)\gamma} \\ & + i g_1 B^\mu \left[\frac{1}{2} \cdot \frac{1}{3}\right] Q'_{(j_6)\gamma} \end{aligned} \quad (36)$$

$$\begin{aligned} \nabla^\mu \psi'_{q(i_6)\alpha} & \equiv \partial^\mu \psi'_{q(i_6)\alpha} + i g_3 G_a^\mu Y_{\alpha\beta}^a \psi'_{q(i_6)\beta} + i g_{sp} A_{\mu j} T_{i_6 j_6}^{(j)} \psi'_{q(j_6)\alpha} \\ & + i g_1 B^\mu \left[\frac{1}{2} \cdot \frac{1}{3}\right] \psi'_{q(i_6)\alpha}. \end{aligned} \quad (37)$$

For the $\mathcal{L}_{superpotential}$ we have the prototype structure

$$\mu\eta_{i_6j_6}\mathcal{H}_{(i_6)}\hat{\Phi}_{(j_6)}\Big|_{\theta\theta} = \mu\eta_{i_6j_6}\left[H_{(i_6)}F_{\phi(j_6)} + \phi_{(j_6)}F_{H(i_6)} - \psi_{H(i_6)}\psi_{\phi(j_6)}\right] \quad (38)$$

$$\begin{aligned} g_{eI}\eta_{i_6j_6}\mathcal{H}_{(i_6)}\mathcal{L}_{(j_6)}\mathcal{R}^I\Big|_{\theta\theta} &= g_{eI}\eta_{i_6j_6}\left[H_{(i_6)}L'_{(j_6)}F_R^I + H_{(i_6)}F_{l(j_6)}R'^I + F_{H(i_6)}L'_{(j_6)}R'^I \right. \\ &\quad \left. - H_{(i_6)}\psi'_{l(j_6)}\psi_R'^I - L'_{(j_6)}\psi_{H(i_6)}\psi_R'^I - \psi_{H(i_6)}\psi'_{l(j_6)}R'^I\right] \end{aligned} \quad (39)$$

3 $B_d^0 \bar{B}_d^0$ mixing in SUSY $Sp(6)$

We now study $B_d^0 \bar{B}_d^0$ mixing in the framework of SUSY $Sp(6)$. In a previous paper [2], we investigated $B_d^0 \bar{B}_d^0$ in the framework of the ordinary (non-SUSY) $Sp(6)$. It was shown that the tree-level contributions due to the extra Z' of the $Sp(6)$ model enhances the FCNC contribution to the mixing parameter x_d . Along the same lines, the gluino contributions of the MSSM, as described in reference [14], also tend to enhance x_d . These, however, proved useful for lower values of the top mass m_t . With the recent experimental limits [5] on m_t of $158 \lesssim m_t \lesssim 194$ GeV, the SM contribution to $B_d^0 \bar{B}_d^0$ mixing maybe high enough to be within the ARGUS, CLEO result [4] of $0.57 \lesssim x_d \lesssim 0.77$. This implies that models which enhance x_d may not be so appealing. Because of the uncertain mass of the Z' , we can always find a value for it to fit the value for x_d . However, with the high m_t , the $m_{Z'}$ becomes uninterestingly large. It turns out that in the SUSY $Sp(6)$ model, the additional contributions of the \tilde{z}' (Z' -ino) tend to cancel the other contributions. This leads to a lower $m_{Z'}$ value. It should be stated, however, that the calculation of x_d involve uncertain parameters that make the conclusions based on the numerics less definitive than we would want to. Nevertheless, the relative numerical relationships can give us definite statements with respect to possible effects.

The SM and $Sp(6)$ model contributions had been discussed in reference [2]. Here, we will focus our discussion on the SUSY contributions. Let us first discuss the analysis of the contributions of the MSSM ¹ to $B_d^0 \bar{B}_d^0$ mixing.

Because of the richer particle spectrum of a SUSY theory, we expect new contributions to FCNC. Of course, these FCNC contributions must not be too additively large for SUSY to remain acceptable.

The three additional contributions to $B_d^0 \bar{B}_d^0$ mixing at the one loop level in the MSSM are due

1. to the physical charged scalar higgs;
2. to the SUSY partner of the W (or more precisely the physical charginos) and the charged scalar higgs and
3. to the neutralinos and gluinos.

For the first contribution in item 2 above, we basically replace the W by its SUSY partner, \tilde{w} and the u , c and t by their SUSY partners \tilde{u} , \tilde{c} and \tilde{t} respectively. Item 3 above on the other hand, is less obvious. It was only realized in 1983 [11] and is unique to SUSY theories since it has no SM analogue. Let us briefly explain why item 3 above is less obvious.

Working in component fields, upon the elimination of the auxiliary fields from the lagrangian of section 2 and with the appropriate $\mathcal{L}_{soft-breaking}$ terms of equation 24, we can calculate the squark mass matrix (see page 3468 of reference [9]). Let us assume for the moment that the mass matrix $(m_Q^2)^{IJ}$ of page 3468 reference [9] is diagonal, i.e. $(m_Q^2)^{IJ} \longrightarrow (m_Q^2)^{IJ} \delta^{IJ}$ (usually this is assumed, see for example equation 2.2 of reference [12]). We can then write the squared mass matrix of the superpartner of the left-handed down squarks as

$$M_{Q_2}^2 = \mu_{Q_2}^{(0)} \mathbf{1} + \mu_{Q_2}^{(1)} M_{qd}^\dagger M_{qd} \quad (40)$$

¹For an excellent paper in which the notation is close to ours on MSSM, see reference [9].

where M_{q_d} is the mass matrix of the down quarks and $\mu_{Q_2}^{(0)}$ and $\mu_{Q_2}^{(1)}$ are some parameter coefficients. Hence, in equation 40, we see that diagonalizing the down quark matrix through a redefinition of fields will automatically diagonalize $M_{Q_2}^2$. Thus a coupling

$$\Lambda_G^a \bar{q}_d^I Q_{(2)}^J \quad (41)$$

where q_d^I and $Q_{(2)}^J$ are the initial down quark and squark fields and Λ_G^a is the gluino, will be diagonalized upon the redefinition of the quark and squark fields since the unitary matrices which are used to redefine them to get their mass eigenstates are the same. Hence, no FCNC occurs. However, when one renormalizes $M_{Q_2}^2$ from its initial value in equation 40 at the superlarge scale down to the m_W scale, it is shown [11] that due to the term $\epsilon_{i_2 j_2} u^{IJ} \mathcal{H}_{(i_2)}^2 \mathcal{Q}_{(j_2)\alpha}^I \mathcal{U}_\alpha^J \Big|_{\theta\theta}$ in the MSSM superpotential (see for example section 5.2.2 reference [17]), there arises a term proportional to the square of the mass matrix ($M_{q_u}^\dagger M_{q_u}$) of the up-type quarks. Equation 40 becomes

$$M_{Q_2}^2 = \mu_{Q_2}^{(0)} \mathbf{1} + \mu_{Q_2}^{(1)} M_{q_d}^\dagger M_{q_d} + \mu_{Q_2}^{(2)} M_{q_u}^\dagger M_{q_u} \quad (42)$$

Hence, diagonalizing $M_{q_d}^\dagger M_{q_d}$ does not diagonalize $M_{Q_2}^2$. This essentially means that the unitary matrices used to redefine the quark fields to get their mass eigenstates will in general be different from the unitary matrices used to redefine the squark fields. Hence, the coupling in equation 41 will not be diagonalized leading to FCNC. The coefficient $\mu_{Q_2}^{(2)}$ is calculated by solving the set of renormalization group equations for the evolution of the SUSY quantities.

It turns out that the contribution due to the gluinos is the most dominant MSSM contribution to $B_d^0 \bar{B}_d^0$ mixing due to the strong coupling α_s ($= \frac{g_3^2}{4\pi}$).

We next discuss the contributions of SUSY $Sp(6)$ to $B_d^0 \bar{B}_d^0$ mixing. For completeness we point out the complete set of (dominant) graphs in SUSY $Sp(6)$ which will contribute to $B_d^0 \bar{B}_d^0$ mixing in figures 1, 2, 3 and 4. Since the standard model, $Sp(6)$ model and the MSSM are all part of SUSY $Sp(6)$, we include their contributions in figures 1, 2 and 3. Let us turn our attention to figures 3 and 4.

To get the physical fields for the down squarks, we use the following approximate equations,

$$Q_{2\alpha}^I = \sum_J V_u^{IJ} Q_{2\alpha}^J \quad (43)$$

$$Q_{1\alpha}^I = \sum_J V_u^{IJ} Q_{1\alpha}^J \quad (44)$$

$$D_\alpha^I = \sum_J V_D^{IJ} D_\alpha^J \quad (45)$$

$$U_\alpha^I = \sum_J V_U^{IJ} U_\alpha^J \quad (46)$$

where V_u , V_D and V_U are the unitary matrices in the MSSM which redefine the initial fermion and sfermion fields to get the corresponding physical fields as in

$$(Q_{(i_2)\alpha}^I, \psi_{q(i_2)\alpha}^I) = \sum_J V_{Q(i_2)}^{IJ} (Q_{(i_2)\alpha}^J, \psi_{q(i_2)\alpha}^J) \quad (47)$$

$$(U_\alpha^I, \psi_{U\alpha}^I) = \sum_J V_U^{IJ} (U_\alpha^J, \psi_{U\alpha}^J) \quad (48)$$

Figure 1: Dominant SM contributions to $B_d^0 \bar{B}_d^0$ mixing

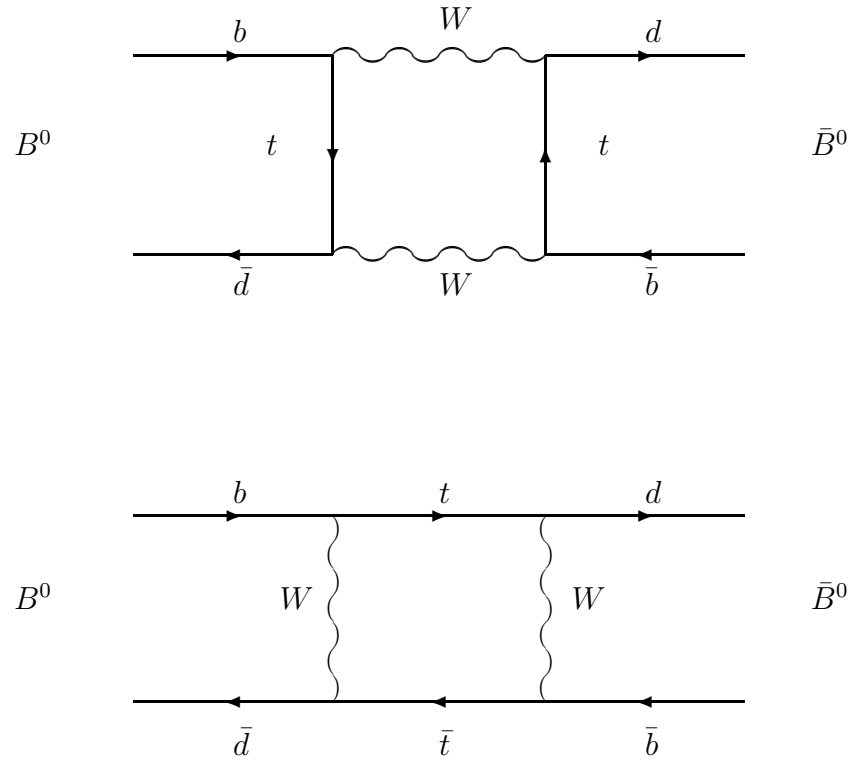


Figure 2: $Sp(6)$ model tree level contributions to $B_d^0 \bar{B}_d^0$ mixing due to the extra Z'

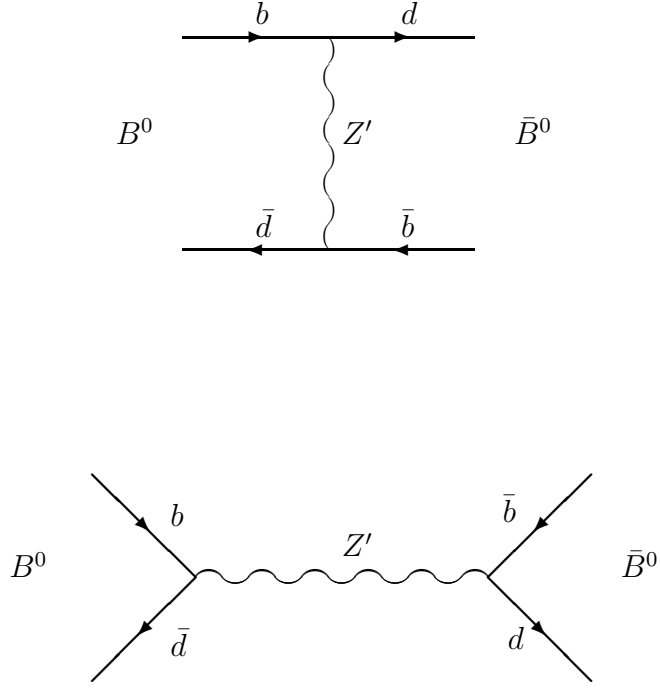


Figure 3: Dominant minimal supersymmetric standard model contributions to $B_d^0 \bar{B}_d^0$ mixing

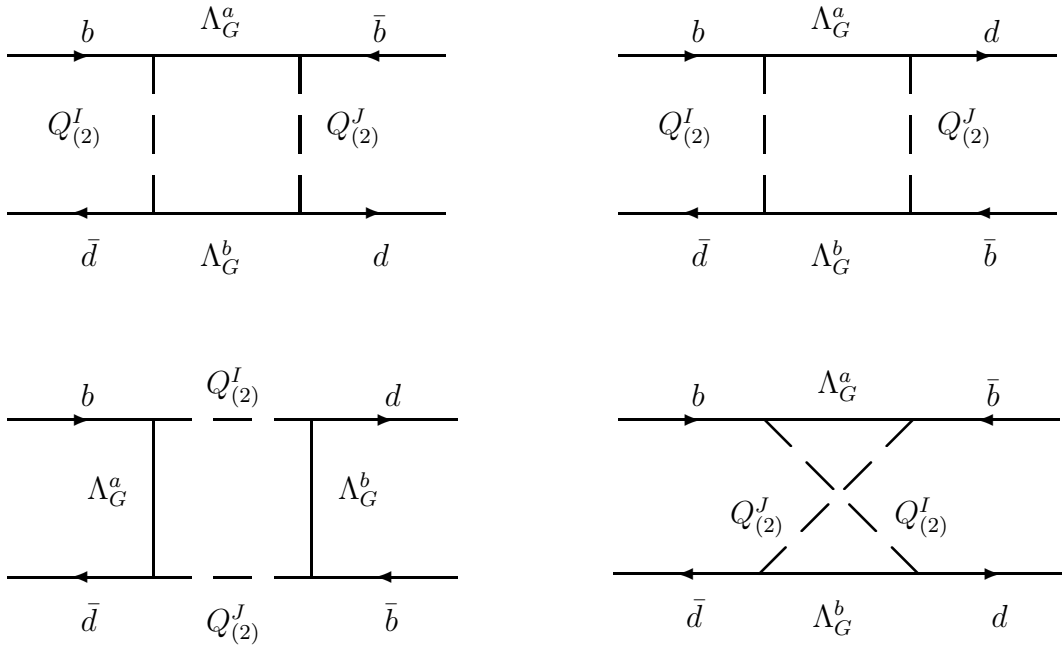
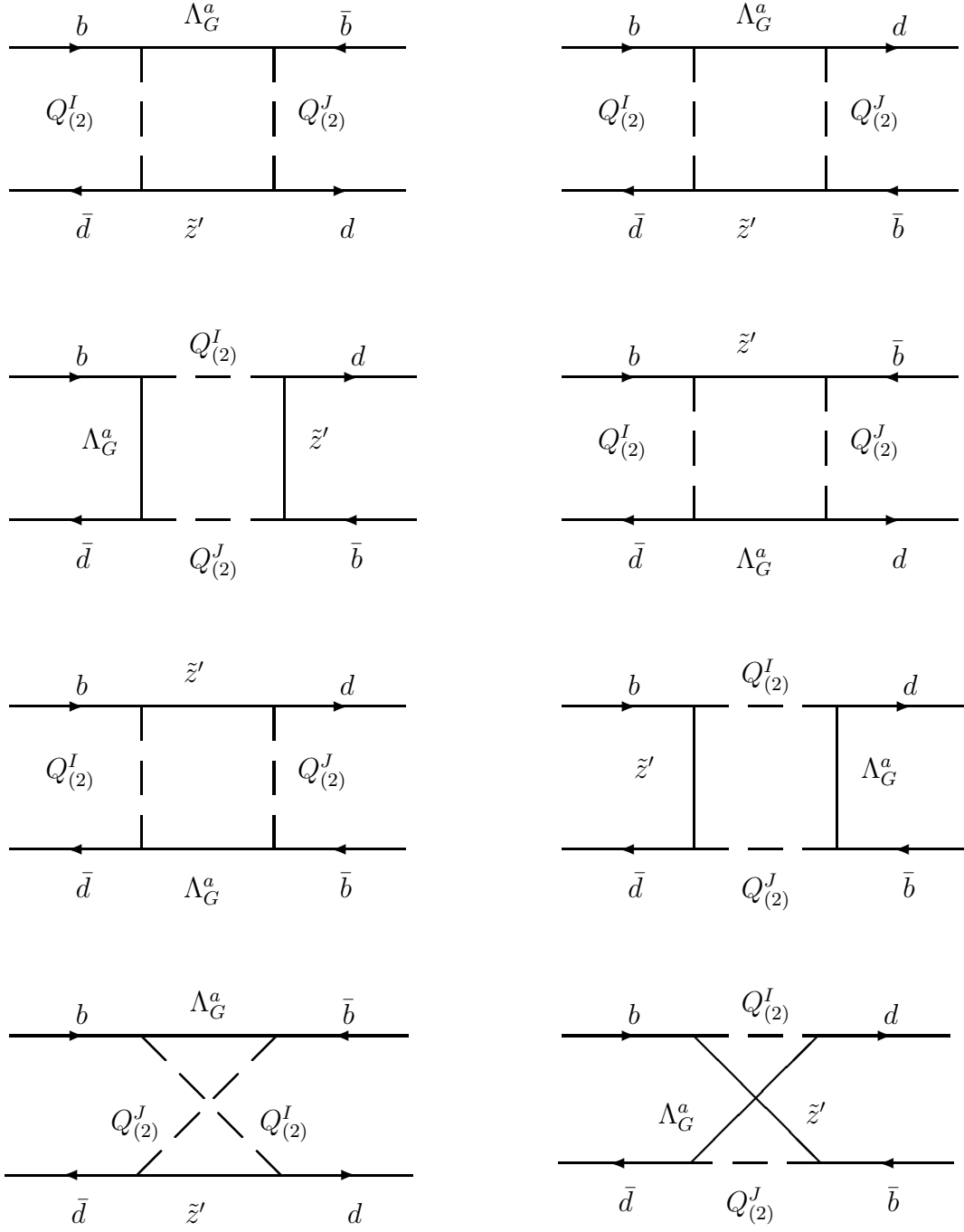


Figure 4: Dominant SUSY $Sp(6)$ contributions to $B_d^0 \bar{B}_d^0$ mixing due to the \tilde{z}' (Z' -ino)



$$(D_\alpha^I, \psi_{D\alpha}^I) = \sum_J V_D^{IJ} (D_\alpha^J, \psi_{D\alpha}^J). \quad (49)$$

Here $i_2 = 1, 2$ for the $\underline{2}$ representation of $SU_L(2)$ and $V_{Q1} = V_u, V_{Q2} = V_d$.

Note that equation 43 is peculiar when compared to equation 47 (with $i_2 = 2$ and $V_{Q(2)} = V_d$) because instead of having V_d , we have V_u . Equations 43 to 46 were derived under the following assumptions:

1. we neglect $Q'_1 - U'$ and $Q'_2 - D'$ mixing;
2. we let $(m_D^2)^{IJ} \longrightarrow (m_D^2)^I \delta^{IJ}, (m_U^2)^{IJ} \longrightarrow (m_U^2)^I \delta^{IJ}$ (no sum in I) and
3. the evolution of the renormalization group equations down to low energies results in

$$M_{Q_2}^2 \simeq \mu_{Q_2}^{(0)} \mathbf{1} + \mu_{Q_2}^{(2)} M_{q_u}^\dagger M_{q_u} \quad (50)$$

$$M_{Q_1}^2 \simeq \mu_{Q_1}^{(0)} \mathbf{1} + \mu_{Q_1}^{(1)} M_{q_u}^\dagger M_{q_u} \quad (51)$$

$$M_D^2 \simeq \mu_D^{(0)} \mathbf{1} \quad (52)$$

$$M_U^2 \simeq \mu_U^{(0)} \mathbf{1} + \mu_U^{(1)} M_{q_u} M_{q_u}^\dagger. \quad (53)$$

Item 1 above is a usual assumption made [13], [14] and [15]. In some low-energy supergravity models, this mixing tends to be small. However, sometimes this assumption maybe inadequate [13].

Item 2 comes from the usual assumption made that all SUSY scalars have degenerate mass [12].

In item 3, we neglected terms proportional to $M_{q_d}^\dagger M_{q_d}$. This is justifiable for some supergravity models where a large top quark mass is responsible for $SU_L(2) \times U_Y(1)$ breaking in the low energy effective theory. [13].

We are also inherently making the approximation in the $Sp(6)$ model that the initial fermion fields are transformed into physical fields by the same transformation matrices V_u, V_d, V_U and V_D which were used in rotating the initial fields to physical fields in the SM as in equations 47 to 49. We justify this by considering the $Sp(6)$ model when it has already spontaneously broken down to the SM.

Similar to the discussion in reference [2], the mixing parameter x_d is

$$x_d \simeq \frac{2 |M_{12}|}{\Gamma} \quad (54)$$

where

$$M_{12} = \langle \bar{B}^0 | \mathcal{H}_{eff} | B^0 \rangle \quad (55)$$

and $\Gamma = \frac{1}{\tau_B}$ where τ_B is the mean lifetime of the B^0 meson.

Adding the contributions due to all the diagrams in figures 1, 2, 3 and 4, we get the following \mathcal{H}_{eff} ,

$$\mathcal{H}_{eff} = F \bar{\Psi}_\alpha^d \gamma_\mu \frac{1}{2} (1 - \gamma^5) \Psi_\alpha^b \bar{\Psi}_\beta^d \gamma^\mu \frac{1}{2} (1 - \gamma^5) \Psi_\beta^b \quad (56)$$

where the operators are understood to be normal ordered and F is defined as

$$\begin{aligned}
F \equiv & \frac{G_F^2 m_t^2 A(z_t)}{4\pi^2 z_t} \left[(V^\dagger)^{13} (V)^{33} \right]^2 \eta_{QCD} + \frac{9\sqrt{2}}{4} G_F \left(\frac{m_W}{m_{Z'}} \right)^2 \left[(V_d^\dagger)^{13} (V_d)^{33} \right]^2 \eta^{sp} \\
& \frac{\alpha_s^2}{36m_{\tilde{g}}^2} \sum_{IJ} (V^\dagger)^{1I} V^{I3} (V^\dagger)^{1J} V^{J3} [4\mathcal{I}_{IJ} + 11\mathcal{K}_{IJ}] \\
& \frac{\sqrt{2}G_F\alpha_s}{24\pi} \left(\frac{m_W}{m_{\tilde{g}}} \right)^2 \sum_{IJ} \left(\frac{m_{Z'}}{m_{\tilde{g}}} \right) \left[V^{I3} V^{J3} \mathcal{V}_{sp}^I \mathcal{V}_{sp}^J + (V^\dagger)^{1I} (V^\dagger)^{1J} \mathcal{T}_{sp}^I \mathcal{T}_{sp}^J \right] \mathcal{I}_{MIJ} \\
& + V^{I3} (V^\dagger)^{1J} \mathcal{V}_{sp}^I \mathcal{T}_{sp}^J \mathcal{K}_{MIJ}
\end{aligned} \tag{57}$$

where V is the CKM matrix as in reference [14]. We will use

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3 \rho e^{i\phi} \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho e^{-i\phi}) & -A\lambda^2 & 1 \end{pmatrix}. \tag{58}$$

In equation 57, G_F is the Fermi constant, m_t is the top mass, $\eta_{QCD} \simeq 0.85$ is the QCD correction for the SM graphs in figure 1, $m_{Z'}$ is the Z' mass, $\alpha_s = \frac{g_s^2}{4\pi} \simeq 0.1134$, $m_{\tilde{g}}$, is the gluino mass, $m_{Z'}$ is the Z' -ino mass and m_W is the W-boson mass. η^{sp} is the QCD correction to the tree level graph in figure 2 and is given by [2],

$$\eta^{sp} = \left[\frac{\alpha_s(m_{Z'}^2)}{\alpha_s(m_t^2)} \right]^{6/21} \left[\frac{\alpha_s(m_t^2)}{\alpha'_s(\mu^2)} \right]^{6/23} \tag{59}$$

where the running strong coupling constant is

$$\alpha_s(Q^2) = \frac{12\pi}{33 - 2n_f} \frac{1}{\ln(\frac{Q^2}{\Lambda^2})}. \tag{60}$$

Note that n_f is the number of quark flavors and α'_s is α_s evaluated in an effective five-quark theory resulting from the step of removing the t-quark from explicitly appearing in the theory.

$\frac{A(z_t)}{z_t}$ is given by [2],

$$\frac{A(z_t)}{z_t} = \frac{1}{4} + \frac{9}{4(1 - z_t)} - \frac{3}{2(1 - z_t)^2} - \frac{3z_t^2 \ln z_t}{2(1 - z_t)^3} \tag{61}$$

where

$$z_t \equiv (m_t/m_W)^2. \tag{62}$$

The functions \mathcal{I}_{IJ} , \mathcal{K}_{IJ} , \mathcal{V}_{sp}^I , \mathcal{T}_{sp}^I , \mathcal{I}_{MIJ} and \mathcal{K}_{MIJ} are given by

$$\mathcal{I}_{IJ} \equiv \frac{1}{z_I - z_J} \left\{ \left[\frac{z_I \ln z_I}{(1 - z_I)^2} + \frac{1}{1 - z_I} \right] - \left[\frac{z_J \ln z_J}{(1 - z_J)^2} + \frac{1}{1 - z_J} \right] \right\} \tag{63}$$

$$\mathcal{K}_{IJ} \equiv \frac{1}{z_I - z_J} \left\{ \left[\frac{z_I^2 \ln z_I}{(1 - z_I)^2} + \frac{1}{1 - z_I} \right] - \left[\frac{z_J^2 \ln z_J}{(1 - z_J)^2} + \frac{1}{1 - z_J} \right] \right\} \tag{64}$$

$$\mathcal{V}_{sp}^I \equiv (V^\dagger)^{1I} - 3 (V_d^\dagger)^{13} V_u^{3I} \quad (65)$$

$$\mathcal{T}_{sp}^I \equiv V^{I3} - 3 (V_u^\dagger)^{I3} V_d^{33} \quad (66)$$

$$\mathcal{I}_{MIJ} \equiv \frac{z_M \ln z_M}{(z_M-1)(z_M-z_I)(z_M-z_J)} + \frac{z_I \ln z_I}{(z_I-1)(z_I-z_J)(z_I-z_M)} + \frac{z_J \ln z_J}{(z_J-1)(z_J-z_I)(z_J-z_M)} \quad (67)$$

$$\mathcal{K}_{MIJ} \equiv \frac{z_M^2 \ln z_M}{(z_M-1)(z_M-z_I)(z_M-z_J)} + \frac{z_I^2 \ln z_I}{(z_M-1)(z_I-z_J)(z_I-z_M)} + \frac{z_J^2 \ln z_J}{(z_J-1)(z_J-z_I)(z_J-z_M)} \quad (68)$$

where

$$z_I \equiv \left(\frac{m_{Q_2}^I}{m_{\tilde{g}}} \right)^2 \quad (69)$$

$$z_M \equiv \left(\frac{m_{\tilde{z}'} }{m_{\tilde{g}}} \right)^2 \quad (70)$$

$m_{Q_2}^I$ here are the down squark masses given by

$$m_{Q_2}^1 = m_{\tilde{d}} = \sqrt{m_b^2 + |\mu_{Q_2}^{(2)}| m_t^2} \quad (71)$$

$$m_{Q_2}^2 = m_{\tilde{s}} = m_{\tilde{d}} \quad (72)$$

$$m_{Q_2}^3 = m_{\tilde{b}} . \quad (73)$$

Note that in deriving the \mathcal{H}_{eff} of equation 56, the gluinos and the \tilde{z}' are majorana spinors. We assign for the four-majorana spinors,

$$\Lambda_G^a = \begin{pmatrix} -i\lambda_G^a \\ i\bar{\lambda}_G^a \end{pmatrix} \quad (74)$$

$$\tilde{z}' = \begin{pmatrix} -i\lambda_{Z'} \\ i\bar{\lambda}_{Z'} \end{pmatrix} \quad (75)$$

Since the gluinos and the Z' -inos are majorana spinors, the Feynman rules needed to deal with them are tricky. There had been a number of papers [7] and [18], which discuss Feynman rules for Majorana spinors. The best and the most recent paper which we used in deriving \mathcal{H}_{eff} in equation 56 is reference [19].

In equation 75, we assume \tilde{z}' to be the physical field. We do this to make the calculations more manageable and to do away with too many arbitrary parameters. Strictly speaking, the formation of this neutralino involves the mixing of the additional neutral higgsinos and gauginos.

We note here that similar to equation 16, we have for the \tilde{z}' in equation 75

$$\lambda_{Z'} = \lambda_{A(18)} \quad (76)$$

i.e. it is a component of the superfield \mathcal{A}_{18} in the lagrangian of section 2.

Using equation 56 in equation 55 we can calculate x_d for SUSY $Sp(6)$ using equation 54. We get

$$x_d = \frac{2}{3} [B_B f_B^2] M_B \tau_{B_d} |F| \quad (77)$$

where F is given by equation 57. In equation 77, we applied the vacuum insertion approximation using the normalization for the $\pi \rightarrow \mu\nu$

$$\langle 0 | \bar{\psi}_u \gamma_\mu \gamma^5 \psi_d | \pi \rangle = \frac{i p_\mu f_\pi}{\sqrt{2 E_p}}. \quad (78)$$

We have the “bag” factor B_B which takes into account all deviations from the vacuum insertion approximation. The quantity f_B is the corresponding f_π for the B meson system. M_B is the mass of the B^0 meson. We note here that coefficients due to color statistics as in reference [16] for the various graphs have been carefully calculated following the discussion of section II of reference [20].

With equation 77, we can demonstrate that the \tilde{z}' contribution can play a significant role in suppressing the mixing parameter x_d to within the experimentally acceptable range $0.57 \lesssim x_d \lesssim 0.77$ [4] especially now that the top quark mass can assume a large value of $158 \lesssim m_t \lesssim 194$ GeV. Using reasonable values of the parameters, $m_{\tilde{g}} = 141$ GeV, $m_{Z'} = 4$ TeV, $m_{\tilde{z}'} = 900$ GeV, we present the plots in figure 5. For the uncertain parameters $B_B f_B^2$ and the CKM matrix's A and ρ , we used the central values. ϕ in the CKM matrix was set to $\frac{\pi}{2}$.

Figure 5: Plot of the $B_d^0 \bar{B}_d^0$ mixing parameter x_d versus the top mass m_t for the SM (solid line), MSSM (dashed-line) and for SUSY $Sp(6)$ (dot-dashed line). The region between the two horizontal lines are the experimentally allowed region from the ARGUS, CLEO result.

The region between the two horizontal lines are the experimentally allowable range for the mixing parameter x_d . Figure 5 seem to indicate that the standard model and the minimal supersymmetric standard model may not be appealing since x_d is a bit too high for the experimentally allowable range for $158 \lesssim m_t \lesssim 194$ GeV.² With the inclusion of the Z' and \tilde{z}' contributions, however, a suppression (mainly due to the \tilde{z}') of x_d occurs which makes it fall well within the allowable range for $158 \lesssim m_t \lesssim 194$ GeV. The \tilde{z}' contribution inherently cancels out the other contributions properly to make x_d fall within the experimentally preferred range. Note that we used $m_{Z'} = 4$ TeV here. It turns out that because of the suppression due to the \tilde{z}' , a $m_{Z'} = 3$ TeV can still yield an x_d within the allowable region for $158 \lesssim m_t \lesssim 194$ GeV. Although in figure 5 we did not include QCD corrections on the SUSY graphs, we expect them to suppress x_d further, allowing for even lighter $m_{Z'}$ of about 2 TeV.

A very interesting feature of the \tilde{z}' contribution is that $m_{\tilde{z}'} < m_{Z'}$ if the cancellation is to be big enough. We have here a model where experiment indicates the possibility of a gaugino with less mass than the gauge boson.

²Of course, these models can still give low values of x_d for some range of the uncertain parameters. However, as indicated above, we used the central values of these uncertain parameters which we feel is the more reasonable thing to do.

On page 637 of reference [21], it was mentioned that neutralinos κ_i^0 in MSSM (actually a subclass of this) contribute negligibly to FCNC amplitudes. Here, however, we have a neutralino, namely \tilde{z}' , which may contribute significantly to FCNC in the form of x_d . The reason is again unique to the presence of the horizontal symmetry. Couplings due to κ_i^0 result to factors $\sim G_F^2$ while for \tilde{z}' one gets factors $\sim G_F \alpha_s$. These arise only if there is a (weak) neutralino and a gluino traversing the loop of the box graph. Since the gluino cannot change flavor, the neutralino must. Hence, only a theory with flavor-changing (weak) neutralinos can have this such as in the SUSY $Sp(6)$ model.

A number of recent papers [22] to [24], discussed flavor (or horizontal) symmetries in SUSY in the context of squark mass degeneracy. One motivation for this is that squark mass degeneracy tends to control large SUSY FCNC effects. In what we have just presented, we can view flavor symmetries in SUSY in another light, and that is, the presence of the SUSY partners (like \tilde{z}') of the extra horizontal gauge bosons (like Z') of a supersymmetric non-abelian horizontal gauge theory may further suppress FCNC in SUSY theories by cancelling the other contributions as demonstrated by the SUSY $Sp(6)$ model.

4 Conclusions and Outlook

The $Sp(6)$ model has been a very interesting model in addressing the generation problem. The presence of the horizontal subgroup $SU_H(3)$ which relates the different generations gives rise to extra gauge bosons, the lightest set of which are the (W'_1, W'_2, Z') . So far phenomenological studies have been concentrated on the Z' effects to FCNC.

Because of the recent renewed interest on SUSY theories, we have presented in this paper a supersymmetric extension of the $Sp(6)$ model (SUSY $Sp(6)$) by writing down a supersymmetric $SU_C(3) \times Sp_L(6) \times U_Y(1)$ gauge invariant lagrangian. Its derivation follows closely that of the extension of the standard model to the minimal supersymmetric standard model. With the introduction of the second type of higgs, we can easily make SUSY $Sp(6)$ anomaly-free as in the MSSM.

As a first step to studying the phenomenological consequences of SUSY $Sp(6)$, we analyzed its dominant contributions to $B_d^0 \bar{B}_d^0$ mixing which include that of the SM, $Sp(6)$ model, MSSM and the contributions due to the Z' -ino (\tilde{z}'). By plotting the mixing parameter x_d (as computed in the SUSY $Sp(6)$ framework) versus m_t (top mass), we are led to the following observations:

1. a cancellation of FCNC effects due to \tilde{z}' ,
2. more pronounced contribution of a weak neutralino in box graphs involving FCNC and
3. a gaugino with a lighter mass than the gauge boson.

The first item can be a reason to possibly view studies of SUSY horizontal symmetries in a new light. Instead of just looking at horizontal symmetries in SUSY as a way to make squark mass degeneracy more natural thereby reducing FCNC, we can also view horizontal symmetries in SUSY with respect to the effects of the Z' -ino which may reduce FCNC as well.

The second item is fairly unique to SUSY theories with horizontal symmetries. Weak neutralinos have usually negligible contributions in box graphs since their coupling introduces G_F^2 whereas gluino graphs introduce α_s^2 . With the weak neutralino \tilde{z}' which may change flavor, box graphs involving these can have bigger contributions $\sim G_F \alpha_s$.

The third item can motivate further studies on SUSY $Sp(6)$ since the $m_{\tilde{z}'} < m_{Z'}$ and thus the effects of \tilde{z}' may be more accessible with respect to the accelerator energies at present.

With the formulation of SUSY $Sp(6)$, one can use the workable lagrangian which we have written down (and which we have checked to reproduce known results in SM, MSSM and $Sp(6)$), to study other phenomenological consequences of a supersymmetric $Sp(6)$ model.

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Appendix

5 Conventions, Notation and Formulas

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (79)$$

$$\eta_{i_6 j_6} = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad (80)$$

$$Q = T_3 + \frac{y}{2} \quad (81)$$

$$\begin{aligned} T^{(1)} &= \frac{1}{2\sqrt{2}}\sigma_1 \otimes \lambda^0 & ; & & T^{(2)} &= \frac{1}{2\sqrt{2}}\sigma_1 \otimes \lambda^1 & ; & & T^{(3)} &= \frac{1}{2\sqrt{2}}\sigma_1 \otimes \lambda^3 \\ T^{(4)} &= \frac{1}{2\sqrt{2}}\sigma_1 \otimes \lambda^4 & ; & & T^{(5)} &= \frac{1}{2\sqrt{2}}\sigma_1 \otimes \lambda^6 & ; & & T^{(6)} &= \frac{1}{2\sqrt{2}}\sigma_1 \otimes \lambda^8 \\ T^{(7)} &= \frac{1}{2\sqrt{2}}\sigma_2 \otimes \lambda^0 & ; & & T^{(8)} &= \frac{1}{2\sqrt{2}}\sigma_2 \otimes \lambda^1 & ; & & T^{(9)} &= \frac{1}{2\sqrt{2}}\sigma_2 \otimes \lambda^3 \\ T^{(10)} &= \frac{1}{2\sqrt{2}}\sigma_2 \otimes \lambda^4 & ; & & T^{(11)} &= \frac{1}{2\sqrt{2}}\sigma_2 \otimes \lambda^6 & ; & & T^{(12)} &= \frac{1}{2\sqrt{2}}\sigma_2 \otimes \lambda^8 \\ T^{(13)} &= \frac{1}{2\sqrt{2}}\sigma_3 \otimes \lambda^0 & ; & & T^{(14)} &= \frac{1}{2\sqrt{2}}\sigma_3 \otimes \lambda^1 & ; & & T^{(15)} &= \frac{1}{2\sqrt{2}}\sigma_3 \otimes \lambda^3 \\ T^{(16)} &= \frac{1}{2\sqrt{2}}\sigma_3 \otimes \lambda^4 & ; & & T^{(17)} &= \frac{1}{2\sqrt{2}}\sigma_3 \otimes \lambda^6 & ; & & T^{(18)} &= \frac{1}{2\sqrt{2}}\sigma_3 \otimes \lambda^8 \\ T^{(19)} &= \frac{1}{2\sqrt{2}}\mathbf{1} \otimes \lambda^2 & ; & & T^{(20)} &= \frac{1}{2\sqrt{2}}\mathbf{1} \otimes \lambda^5 & ; & & T^{(21)} &= \frac{1}{2\sqrt{2}}\mathbf{1} \otimes \lambda^7 \end{aligned} \quad (82)$$

$$[Y^a, Y^b] = if_{abc} Y^c, \text{ where } Y^a = \frac{\lambda^a}{2} = SU_C(3) \text{ generators in } \underline{\mathbf{3}} \text{ representation} \quad (83)$$

$$\bar{Y}^a = SU_C(3) \text{ generators in } \bar{\underline{\mathbf{3}}} \text{ representation } \ni (\bar{Y}^a)_{\beta\alpha} = (-Y^a)_{\alpha\beta} \quad (84)$$

$$i, j, k = 1, 2, \dots, 21 = Sp_L(6) \text{ generator indices unless specified otherwise} \quad (85)$$

$$i_6, j_6, k_6 = 1, 2, \dots, 6 = \text{indices for the } \underline{\mathbf{6}} \text{ representation of } Sp_L(6) \quad (86)$$

$$I, J, K = 1, 2, 3 = \text{generation indices} \quad (87)$$

$$\alpha, \beta, \gamma = 1, 2, 3 = \text{indices for the } \underline{\mathbf{3}} \text{ representation of } SU_C(3) \quad (88)$$

$$a, b, c = 1, 2, \dots, 8 = SU_C(3) \text{ generator indices} \quad (89)$$

$$g_1, g_2, g_3 = U_Y(1), SU_L(2), SU_C(3) \text{ gauge couplings respectively} \quad (90)$$

$$B^\nu, A_i^\nu, G_a^\mu \equiv U_Y(1), Sp_L(6), SU_C(3) \text{ gauge bosons respectively} \quad (91)$$

$$\Psi_L \equiv \frac{1}{2}(1 - \gamma_5)\Psi \ ; \ \Psi_{rt} \equiv \frac{1}{2}(1 + \gamma_5)\Psi \quad (92)$$

$$\begin{aligned} \Psi'_{(1)L} &= \nu'_{eL} = \nu_{eL}^1 \\ \Psi'_{(2)L} &= \nu'_{\mu L} = \nu_{eL}^2 \\ \Psi'_{(3)L} &= \nu'_{\tau L} = \nu_{eL}^3 \\ \Psi'_{(4)L} &= e'_L = e_L^1 \\ \Psi'_{(5)L} &= \mu'_L = e_L^2 \\ \Psi'_{(6)L} &= \tau'_L = e_L^3 \end{aligned} \quad (93)$$

$$\begin{aligned} (\Psi'_Q)_{(1)\alpha L} &= u'_{\alpha L} = u_{\alpha L}^1 \\ (\Psi'_Q)_{(2)\alpha L} &= c'_{\alpha L} = u_{\alpha L}^2 \\ (\Psi'_Q)_{(3)\alpha L} &= t'_{\alpha L} = u_{\alpha L}^3 \\ (\Psi'_Q)_{(4)\alpha L} &= d'_{\alpha L} = d_{\alpha L}^1 \\ (\Psi'_Q)_{(5)\alpha L} &= s'_{\alpha L} = d_{\alpha L}^2 \\ (\Psi'_Q)_{(6)\alpha L} &= b'_{\alpha L} = d_{\alpha L}^3 \end{aligned} \quad (94)$$

$$\Psi_{rt}^1 = e_R, \ \Psi_{rt}^2 = \mu_R, \ \Psi_{rt}^3 = \tau_R \quad (95)$$

$$(\Psi_u)^I_{\alpha rt}, \ni (\Psi_u)^1_{\alpha rt} = u_{\alpha R}, \ (\Psi_u)^2_{\alpha rt} = c_{\alpha R}, \ (\Psi_u)^3_{\alpha rt} = t_{\alpha R} \quad (96)$$

$$(\Psi_d)^I_{\alpha rt}, \ni (\Psi_d)^1_{\alpha rt} = d_{\alpha R}, \ (\Psi_d)^2_{\alpha rt} = s_{\alpha R}, \ (\Psi_d)^3_{\alpha rt} = b_{\alpha R} \quad (97)$$

$$\mathcal{D}^{(1)\mu}\Psi_{rt}^I \equiv \partial^\mu \Psi_{rt}^I + i\frac{g_1}{2}B^\mu y \Psi_{rt}^I \quad (98)$$

$$\mathcal{D}^{(2)\mu}\Psi'_{(i_6)L} \equiv \partial^\mu \Psi'_{(i_6)L} + ig_{sp}A_j^\mu T_{i_6 j_6}^{(j)} \Psi'_{(j_6)L} + i\frac{g_1}{2}B^\mu y \Psi'_{(i_6)L} \quad (99)$$

$$\begin{aligned} \nabla^\mu (\Psi'_Q)_{(i_6)\alpha L} &\equiv \partial^\mu (\Psi'_Q)_{(i_6)\alpha L} + ig_3 G_a^\mu Y_{\alpha\beta}^a (\Psi'_Q)_{(i_6)\beta L} + ig_{sp} A_j^\mu T_{i_6 j_6}^{(j)} (\Psi'_Q)_{(j_6)\alpha L} \\ &\quad + i\frac{g_1}{2} B^\mu y (\Psi'_Q)_{(i_6)\alpha L} \end{aligned} \quad (100)$$

$$\mathcal{D}^{(2)\mu}(\Psi_u)^I_{\alpha rt} \equiv \partial^\mu (\Psi_u)^I_{\alpha rt} + ig_3 G_a^\mu \bar{Y}_{\alpha\beta}^a (\Psi_u)^I_{\beta rt} + i\frac{g_1}{2} B^\mu y \delta_{\alpha\beta} (\Psi_u)^I_{\beta rt} \quad (101)$$

$$\mathcal{D}^{(2)\mu}(\Psi_d)^I_{\alpha rt} \equiv \partial^\mu (\Psi_d)^I_{\alpha rt} + ig_3 G_a^\mu \bar{Y}_{\alpha\beta}^a (\Psi_d)^I_{\beta rt} + i\frac{g_1}{2} B^\mu y \delta_{\alpha\beta} (\Psi_d)^I_{\beta rt} \quad (102)$$

$$(\mathcal{D}_\mu \lambda_G)_a = \partial_\mu \lambda_{Ga} - g_3 f_{abc} G_{\mu b} \lambda_{Gc} \quad (103)$$

$$W_{G\eta} = -\frac{1}{4} \bar{D} \bar{D} e^{-2g_3 Y^a \mathcal{G}_a} D_\eta e^{2g_3 Y^b \mathcal{G}_b}, \quad \text{for } SU_C(3) \quad (104)$$

$$W_{A\eta} = -\frac{1}{4}\bar{D}\bar{D}e^{-2g_{sp}T^{(i)}\mathcal{A}_i}D_{\eta}e^{2g_{sp}T^{(i)}\mathcal{A}_i}, \quad \text{for } Sp_L(6) \quad (105)$$

$$W_{B\eta} = -\frac{1}{4}\bar{D}\bar{D}e^{-\hat{\mathcal{B}}}D_{\eta}e^{\hat{\mathcal{B}}} = -\frac{1}{4}\bar{D}\bar{D}D_{\eta}\hat{\mathcal{B}}, \quad \text{for } U_Y(1) \quad (106)$$